

Geometric massive higher spins and current exchanges

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Generalised Fierz-Pauli mass terms allow to describe massive higher-spin fields on flat background by means of simple quadratic deformations of the corresponding geometric, massless Lagrangians. In this framework there is no need for auxiliary fields. We briefly review the construction in the bosonic case and study the interaction of these massive fields with external sources, computing the corresponding propagators. In the same fashion as for the massive graviton, but differently from theories where auxiliary fields are present, the structure of the current exchange is completely determined by the form of the mass term itself.

Based on the talk presented at the 3rd RTN-Forces Universe Workshop - Valencia, Spain, 1-5 October 2007

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1 Introduction and summary

In this contribution we would like to briefly review the work [1], where we proposed massive Lagrangians for higher-spin fields from a perspective such that, in particular, no need for auxiliary fields emerges. In addition, we compute here the propagator of those theories, along the lines of the extensive analysis of similar issues performed in [2] and [3].

As an introduction, let us recall that the Lagrangian description of massive, lower-spin fields¹ is both simple and unique. For instance, in the spin 2 case, the Fierz-Pauli Lagrangian [4],

$$\mathcal{L} = \frac{1}{2} h^{\mu\nu} \{ \mathcal{R}_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} \mathcal{R} - m^2 (h_{\mu\nu} - \eta_{\mu\nu} h^\alpha{}_\alpha) \}, \quad (1)$$

where $\mathcal{R}_{\mu\nu}$ and \mathcal{R} indicate the linearised Ricci tensor and Ricci scalar respectively, describes the *only* consistent quadratic deformation of the linearised Einstein-Hilbert theory [5]. Actually, for all bosonic and fermionic lower-spin fields, a consistent and unique massive theory can be obtained adding to their massless Lagrangians suitable quadratic terms.

By contrast, the traditional description of massive higher-spin fields² is neither as simple, nor it is unique, and in particular, for spin $s \geq \frac{5}{2}$, auxiliary fields are usually found to be needed off-shell. As already noticed in [4] they can be chosen in several ways, so that the theory looses uniqueness (although a minimal choice was first identified in [7]³), while the resulting Lagrangians do not look like simple quadratic deformations of the corresponding massless ones.

The key observation in order to understand the origin of this difference between lower- and higher-spin fields is to notice that, still focusing on the example of spin 2, consistency of the dynamics described by (1) is guaranteed by the Bianchi identity satisfied by the Einstein tensor,

$$\partial^\alpha \{ \mathcal{R}_{\alpha\mu} - \frac{1}{2} \eta_{\alpha\mu} \mathcal{R} \} \equiv 0, \quad (2)$$

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¹ i.e. fields with spin $s \leq 2$.

² For reviews on the subject of higher-spin gauge fields see [6].

³ To describe spin- s massive degrees of freedom in [7] a symmetric, traceless rank- s tensor was introduced, together with a set of symmetric and traceless auxiliary tensors of rank $s-2, s-3, \dots, 0$. In fact, as already noticed in [8], by means of suitable field redefinitions it is possible to collect all those fields in a set of only *two*, *traceful* symmetric tensors of rank s and $s-3$ respectively.

which, in its turn, reflects the geometrical underpinnings of the massless sector of the theory. Implementing this piece of information on-shell allows to recover the Fierz-Pauli constraint

$$\partial^\alpha h_{\alpha\mu} - \partial_\mu h^\alpha_\alpha = 0, \quad (3)$$

which is ultimately responsible for the reduction of the equations of motion to the system

$$\begin{aligned} (\square - m^2) h_{\mu\nu} &= 0, \\ \partial^\alpha h_{\alpha\mu} &= 0, \\ h^\alpha_\alpha &= 0, \end{aligned} \quad (4)$$

describing the irreducible propagation of massive, spin-2 degrees of freedom. On the other hand, in the (Fang-)Fronsdal theory of massless higher-spin fields [9] the corresponding Einstein tensors *are not fully divergenceless*, as a reflection of their lack of direct geometrical meaning. In fact, whereas the higher-spin curvatures introduced by de Wit and Freedman in [10] are *fully* gauge-invariant under the abelian gauge transformation of the potential⁴,

$$\delta \varphi_{\mu_1 \dots \mu_s} = \partial_{\mu_1} \Lambda_{\mu_2 \dots \mu_s} + \dots, \quad (5)$$

without any conditions on $\Lambda_{\mu_1 \dots \mu_{s-1}}$, it turns out that the basic equation of the Fronsdal theory,

$$\mathcal{F}_{\mu_1 \dots \mu_s} \equiv \square \varphi_{\mu_1 \dots \mu_s} - \partial_{\mu_1} \partial^\alpha \varphi_{\alpha \mu_2 \dots \mu_s} + \dots + \partial_{\mu_1} \partial_{\mu_2} \varphi^\alpha_{\alpha \mu_3 \dots \mu_s} + \dots = 0, \quad (6)$$

is gauge-invariant under (5) only if the parameter is taken to be *traceless*: $\Lambda^\alpha_{\alpha \mu_3 \dots \mu_{s-1}} \equiv 0$, a condition which indicates that such a theory as it stands cannot have a direct geometrical interpretation. Moreover, as a consequence of the constraint on the trace of the gauge parameter, the corresponding Einstein tensor need not be (and is not) identically divergenceless, and for this reason, as discussed in [11], in order to get a consistent massive theory, a simple quadratic deformation of the Fronsdal Lagrangians alone is not enough, and auxiliary fields are to be introduced⁵.

A *geometric* description of massless higher-spin gauge fields, where all quantities of dynamical interest are actually built from curvatures, was proposed in [12, 13, 2] for the case of symmetric tensors⁶. The main outcome of the full construction is that, out of infinitely many geometric Lagrangians available at the free level, consistency with the coupling to an external source requires the theory to have a *unique* form. In particular the Einstein tensor of this theory, in the compact notation⁷ of those works, that we shall also exploit here, can be written

$$\mathcal{E}_\varphi = \mathcal{A}_\varphi - \frac{1}{2} \eta \mathcal{A}'_\varphi + \eta^2 \mathcal{B}_\varphi. \quad (7)$$

The generalised Ricci tensor \mathcal{A}_φ , which is fully constructed out of curvatures [2], admits a particularly simple interpretation when written in terms of the Fronsdal tensor \mathcal{F} :

$$\mathcal{A}_\varphi = \mathcal{F} - 3 \partial^3 \gamma_\varphi, \quad (8)$$

⁴ Dots indicate symmetrization of the s indices.

⁵ See [20] for a more recent approach.

⁶ Generalisations to the case of mixed-symmetry gauge fields have been given in [14].

⁷ All symmetrised indices are implicit, and symmetrization without factors among indices is always understood in the product of different tensors. η is the “mostly-plus” space-time metric in d dimensions, “primes”, as well as numbers in square brackets, denote traces while divergences are denoted by “ $\partial \cdot$ ”. Useful combinatorial identities are

$$\begin{aligned} (\partial^p \varphi)' &= \square \partial^{p-2} \varphi + 2 \partial^{p-1} \partial \cdot \varphi + \partial^p \varphi', & \partial^p \partial^q &= \binom{p+q}{p} \partial^{p+q}, \\ (\eta^k \varphi)' &= [D + 2(s+k-1)] \eta^{k-1} \varphi + \eta^k \varphi', & \eta \eta^{n-1} &= n \eta^n. \end{aligned}$$

where γ_φ is a *non-local* tensor⁸, transforming under (5) with the trace of the gauge parameter:

$$\delta \gamma_\varphi = \Lambda'. \quad (9)$$

This implies that, with the same gauge fixing, it is possible both to remove all non localities from the equations of motion and to recover the Fronsdal form (6)⁹, thus showing the consistency of the construction. It should be stressed that all non-localities can be removed also *off-shell*, and without performing any gauge-fixing, at the price of introducing *auxiliary fields*. Indeed, whereas an unconstrained description of the Fronsdal dynamics involving auxiliary fields was already known since some time [15], the form (8) of the tensor \mathcal{A}_φ suggests a very economical alternative option, first proposed in [13, 16], that is to substitute the non-local tensor γ_φ with a *compensator* field α having the same gauge transformation:

$$\delta \alpha = \Lambda'. \quad (10)$$

In this way the *local* tensor

$$\mathcal{A} = \mathcal{F} - 3 \partial^3 \alpha, \quad (11)$$

can be used as a starting point to build local, unconstrained Lagrangians [17], whose completion only requires a further auxiliary field β , with the transformation property $\delta \beta = \partial \cdot \partial \cdot \partial \cdot \Lambda$. Finally, the elimination of the higher derivatives appearing in connection with the compensator α can again be implemented in a rather economical fashion, thus leading to an ordinary derivative unconstrained Lagrangian involving a total of five fields for any spin, as described in [1] (see also [18] for related work).

For our present purposes, the main feature of the tensor (7) (and actually of all tensors among the infinitely many available at the free level) is to be *identically divergenceless*, as required by the absence of constraints on the gauge parameter. It is then possible to look for quadratic deformations of the corresponding class of Lagrangians in the spirit of the Fierz-Pauli description of the massive graviton leading to (1). The concrete realization of this program was the main result of [1] where generalised Fierz-Pauli mass terms were proposed, both for bosons and fermions of any spin. Their form, for the simple cases of spin 4 and spin $\frac{7}{2}$, looks

$$M_\varphi = \varphi_{\mu_1 \dots \mu_4} - (\eta_{\mu_1 \mu_2} \varphi_{\alpha \mu_3 \mu_4}^\alpha + \dots) - (\eta_{\mu_1 \mu_2} \eta_{\mu_3 \mu_4} + \dots) \varphi_{\alpha \beta}^{\alpha \beta}, \quad (12)$$

$$M_\psi = \psi_{\mu_1 \mu_2 \mu_3} - (\gamma_{\mu_1} \gamma^\alpha \psi_{\alpha \mu_2 \mu_3} + \dots) - (\eta_{\mu_1 \mu_2} \psi_{\alpha \mu_3}^\alpha + \dots) - (\gamma_{\mu_1} \eta_{\mu_2 \mu_3} + \dots) \gamma^\alpha \psi_{\alpha \beta}^\beta, \quad (13)$$

where it is already possible to appreciate one basic feature of the general result: all (gamma-)traces of the field enter the mass terms, in a sequence starting with the Fierz-Pauli contributions. We shall review the derivation of the generalised Fierz-Pauli mass terms in Section 2, restricting our attention to bosons.

In [2, 3] a systematic analysis of the interaction between unconstrained higher-spin fields and external sources, both in flat and in (A)dS backgrounds, was proposed. This provided first of all a test of the consistency of the unconstrained Lagrangians, both in the geometric, non-local framework of [12, 13, 2] and in its minimal local counterpart of [17, 2], *versus* the structure of the corresponding propagators. Together with that, a few interesting issues were also addressed, such as the persistence of the vDVZ discontinuity [19] for all spins on flat backgrounds [2, 3], and its disappearance on (A)dS spaces [3].

Here we would like to supplement the results obtained in [2, 3], computing the current exchange for the massive Lagrangians of [1], whose consistency can then be compared with the result obtained in the local setting by Kaluza-Klein reduction in [2, 3]. This is done in Section 3.

An interesting aspect of this analysis relies in the fact that, under the conditions of conservation of the sources, *the full structure of the massive propagator is encoded in the coefficients of the mass terms*. This

⁸ Since for spin s the curvatures of [10] contain s derivatives, in order for the differential operator appearing in (8) to carry the same dimensions as the D'Alembertian operator, non-localities are to be introduced as an unavoidable intermediate feature of the geometric construction.

⁹ This comes from the fact that the Lagrangian equations $\mathcal{E}_\varphi = 0$ can be shown to imply $\mathcal{A}_\varphi = 0$.

is already true for the massive graviton, where it represents a clue to understand the “rigidity” of the Fierz-Pauli mass term of (1), while in our framework this feature provides a rather strong consistency check on the form of the generalised Fierz-Pauli mass term proposed in [1].

2 Generalised Fierz-Pauli mass terms

In this Section we would like to summarise the construction of the generalised Fierz-Pauli mass terms given in [1], focusing for simplicity on the bosonic case¹⁰. For definiteness, we assume that the massless sector of the theory is described by the divergenceless Einstein tensor (7), with the generalised Ricci tensor \mathcal{A}_φ given by (8)¹¹. We thus look for a massive Lagrangian for higher-spin bosons of the form

$$\mathcal{L} = \frac{1}{2} \varphi \{ \mathcal{E}_\varphi - m^2 M_\varphi \}, \quad (15)$$

where M_φ is a linear function of φ to be determined. The general idea is that M_φ should be a combination of *all* the traces of φ , as expected in our unconstrained setting. Moreover, given that the divergence of the equation of motion

$$\partial \cdot \{ \mathcal{E}_\varphi - m^2 M_\varphi \} = 0, \quad (16)$$

reduces to

$$\partial \cdot M_\varphi = 0, \quad (17)$$

it is clear that the issue at stake is to understand what conditions should be deduced from (17). From the conceptual viewpoint, the main result of [1] was to prove that (17) should imply for all spins the Fierz-Pauli constraint

$$\partial \cdot \varphi - \partial \varphi' = 0, \quad (18)$$

since this condition reveals itself to be necessary and sufficient to recover the irreducibility conditions (4), generalised to a rank- s tensor. To have an idea of how the procedure works, let us discuss in some detail, for the example of spin 4, both the relevance of (18) and the corresponding solution for M_φ . We shall then show how to compute M_φ for the spin- s case, exploiting the requirement that (17) imply (18).

2.1 Spin 4

Let us consider the Lagrangian,

$$\mathcal{L} = \frac{1}{2} \varphi \{ \mathcal{A}_\varphi - \frac{1}{2} \eta \mathcal{A}'_\varphi + \mathcal{B}_\varphi - m^2 M_\varphi \}, \quad (19)$$

where the explicit form of \mathcal{A}_φ in terms of the Fronsdal tensor \mathcal{F} defined in (6) is given by

$$\begin{aligned} \mathcal{A}_\varphi &= \mathcal{F} - 3 \partial^3 \gamma_\varphi, \\ \gamma_\varphi &= \frac{1}{3 \square^2} \partial \cdot \mathcal{F}' - \frac{1}{3} \frac{\partial}{\square^3} \partial \cdot \partial \cdot \mathcal{F}' + \frac{1}{12} \frac{\partial}{\square^2} \mathcal{F}'', \end{aligned} \quad (20)$$

¹⁰ For the corresponding discussion for fermions, together with an account of the fermionic geometry underlying their massless Lagrangians see [1].

¹¹ Let us stress that the non local compensator tensor γ_φ in (8) must have a very specific form. In fact, as shown in [2], there are infinitely many non local tensors displaying the same gauge transformation as γ_φ , each associated to a theory with the correct classical behaviour at free level, and in particular to a divergenceless Einstein tensor, but possessing in general the wrong propagator. What allows to select the (unique) correct form of γ_φ is either the request that the identity

$$\partial \cdot \mathcal{A}_\varphi - \frac{1}{2} \partial \mathcal{A}'_\varphi \equiv 0, \quad (14)$$

(crucial to ensure that the massless propagator have the correct structure) be satisfied, or -equivalently- that the tensor \mathcal{A}_φ be identically doubly traceless.

while \mathcal{B}_φ is fixed by the requirement that $\partial \cdot \mathcal{E}_\varphi = 0$, implying

$$\partial \mathcal{B}_\varphi = \frac{1}{2} \partial \cdot \mathcal{A}'_\varphi. \quad (21)$$

For the mass term M_φ we choose the general combination

$$M_\varphi = \varphi + a \eta \varphi' + b \eta^2 \varphi''. \quad (22)$$

where the coefficients a and b should be chosen in such a way to (17) imply $\mathcal{A}'_\varphi = 0$. On the other hand, \mathcal{A}'_φ starts with \mathcal{F}' together with terms containing at least one divergence of \mathcal{F}' , and from the explicit form of \mathcal{F}'

$$\mathcal{F}' = 2 \square \varphi' - 2 \partial \cdot \partial \cdot \varphi + \partial \partial \cdot \varphi' + \partial^2 \varphi'', \quad (23)$$

we see that the first two terms cannot be compensated by anything in the remainder of \mathcal{A}'_φ , unless the combination

$$\square \varphi' - \partial \cdot \partial \cdot \varphi \quad (24)$$

result to be expressible in terms of higher traces and divergences of φ , as a consequence of the equations of motion. This kind of condition is indeed implemented by the Fierz-Pauli constraint, but would not hold if we had a more general condition of the form $\partial \cdot \varphi - k \partial \varphi' = 0$, with $k \neq 1$, thus showing the very peculiar role played by (18). If we then assume to have fixed the coefficients a and b so that $\partial \cdot M_\varphi = 0$ implies (18), it is possible to show that the following consequences hold:

$$\begin{aligned} \mathcal{F} &= \square \varphi - \partial^2 \varphi', \\ \mathcal{F}' &= 3 \partial^2 \varphi'', \end{aligned} \quad (25)$$

which, in their turn, can be shown to imply $\mathcal{A}'_\varphi = 0$. Consequently, the Lagrangian equation reduces on-shell to the form

$$\mathcal{A}_\varphi - m^2 M_\varphi = 0, \quad (26)$$

where the proper solution for M_φ such as to guarantee that $\partial \cdot M_\varphi = 0$ imply (18), together with its consistency condition $\partial \cdot \varphi' = -\partial \varphi''$ is

$$M_\varphi = \varphi - \eta \varphi' - \eta^2 \varphi''. \quad (27)$$

From the double trace of (26) one obtains $\varphi'' = 0$, which implies $\varphi' = 0$ and finally $(\square - m^2) \varphi = 0$, as required.

2.2 Spin s

In the general case, as already stressed, all traces of φ are expected to contribute to M_φ , so that, for $s = 2n$ or $s = 2n + 1$, its general form would be

$$M_\varphi = \varphi + b_1 \eta \varphi' + b_2 \eta^2 \varphi'' + \dots + b_k \eta^k \varphi^{[k]} + \dots + b_n \eta^n \varphi^{[n]}, \quad (28)$$

where $n = \lfloor \frac{s}{2} \rfloor$. The same argument seen for spin 4 applies also in this case: we would like to obtain $\mathcal{A}'_\varphi = 0$ as a consequence of $\partial \cdot M_\varphi = 0$. To this end we look for coefficients b_1, \dots, b_n such that the divergence of (28) imply (18) *together with its consistency conditions*

$$\partial \cdot \varphi^{[k]} = -\frac{1}{2k-1} \partial \varphi^{[k+1]}, \quad k = 1 \dots n, \quad (29)$$

since it is possible to show that if the latter equations are satisfied then $\mathcal{A}'_\varphi = 0$. More explicitly, if we write the divergence of M_φ in the form

$$\partial \cdot M_\varphi = \partial \cdot \varphi + b_1 \partial \varphi' + \dots + \eta^k (b_k \partial \cdot \varphi^{[k]} + b_{k+1} \partial \varphi^{[k+1]}) + \dots, \quad (30)$$

and we define $\mu_\varphi \equiv \partial \cdot \varphi - \partial \varphi'$, then we would like to rearrange (30) as

$$\partial \cdot M_\varphi = \mu_\varphi + \lambda_1 \eta \mu'_\varphi + \dots + \lambda_k \eta^{[k]} \mu_\varphi^{[k]} + \dots \quad (31)$$

In this fashion, subsequent traces of (31) would imply $\mu_\varphi^{[k]} = 0$, for $k = n, n-1, \dots$ and then finally $\mu_\varphi = 0$, as desired¹². The form of μ_φ immediately fixes the first coefficient to be $b_1 = -1$, whereas consistency with (31) requires

$$\begin{aligned} \lambda_k &= -\frac{b_k}{2k-1}, \\ b_{k+1} &= \frac{b_k}{2k-1}, \end{aligned} \quad (32)$$

whose unique solution is

$$b_{k+1} = -\frac{1}{(2k-1)!!}. \quad (33)$$

We obtain in this way the complete form of the generalised Fierz-Pauli mass term:

$$M_\varphi = \varphi - \eta \varphi' - \eta^2 \varphi'' - \frac{1}{3} \eta^3 \varphi''' - \dots - \frac{1}{(2k-3)!!} \eta^k \varphi^{[k]} - \dots \quad (34)$$

Once the equations of motion are reduced to the form $\mathcal{A}_\varphi - m^2 M_\varphi = 0$, then all traces of φ can subsequently shown to vanish, thus leading to the conclusion that the Lagrangian equations obtained by (15) imply the system $(\square - m^2) \varphi = 0$, $\partial \cdot \varphi = 0$, $\varphi' = 0$, and thus provide a consistent description of massive higher-spin degrees of freedom.

3 Interaction with external sources

One simple possibility to study massive higher-spin fields in a d -dimensional flat background is to deduce their properties from those of the corresponding massless theory in $d+1$ dimensions, subject to a standard procedure of Kaluza-Klein reduction. In this fashion, starting from the massless, unconstrained local Lagrangians of [17, 2], it was possible in [2, 3] to compute the massive propagator for a spin- s field coupled to a conserved source. The result is

$$(p^2 - M^2) \mathcal{J} \cdot \varphi = \sum_{n=0}^{\lfloor \frac{s}{2} \rfloor} \rho_n(d-1, s) \frac{s!}{n! (s-2n)! 2^n} \mathcal{J}^{[n]} \cdot \mathcal{J}^{[n]}. \quad (35)$$

where the coefficients $\rho_n(d-1, s)$ are given by

$$\rho_n(d-1, s) = (-1)^n \prod_{k=1}^n \frac{1}{d-1+2(s-k-1)}. \quad (36)$$

We would like to compare this result with the propagator of the theory defined by the Lagrangian

$$\mathcal{L} = \frac{1}{2} \varphi \{ \mathcal{A}_\varphi - \frac{1}{2} \eta \mathcal{A}'_\varphi + \eta^2 \mathcal{B}_\varphi - m^2 M_\varphi \} - \varphi \cdot \mathcal{J}, \quad (37)$$

¹² This is of course true given that the coefficients λ_k do not imply any *identical* cancellations among the traces of $\partial \cdot M_\varphi$.

where M_φ is given by (34), and the field φ is coupled to a current which we assume to be conserved¹³. Under this condition the divergence of the equations of motion implies the same consequences as for the free case, and the Lagrangian equation reduces to

$$\mathcal{A}_\varphi - m^2 (\varphi - \eta \varphi' - \dots - \frac{1}{(2k-3)!!} \eta^k \varphi^{[k]} \dots) = \mathcal{J}. \quad (38)$$

Successive traces of this last equation, taking into account that under the assumed conditions $\mathcal{A}'_\varphi = 0$, give the following system

$$\begin{aligned} \varphi' + \eta \varphi'' + \eta^2 \frac{1}{3} \varphi^{[3]} \dots + \frac{1}{(2n-3)!!} \eta^{n-1} \varphi^{[n]} &= -\frac{\rho_1}{m^2} \mathcal{J}', \\ \varphi'' + \dots + \frac{1}{(2n-3)!!} \eta^{n-2} \varphi^{[n]} &= +\frac{\rho_2}{m^2} \mathcal{J}'', \\ \dots \dots, \\ \sum_{k=l}^n \frac{\eta^{k-l}}{(2l-3)!!} \varphi^{[l]} &= (-1)^l \frac{\rho_l}{m^2} \mathcal{J}^{[l]}, \\ \dots, \\ \frac{1}{(2n-3)!!} \varphi^{[n]} &= (-1)^n \frac{\rho_n}{m^2} \mathcal{J}^{[n]}, \end{aligned} \quad (39)$$

where we recall that $n = [\frac{s}{2}]$. It is remarkable that the coefficient of $\varphi^{[k]}$ is the same in throughout the system, for all k . In this sense, each of the l.h.s. in (39) really looks like a “right-truncation” of the l.h.s. of (38). Of course this is not strictly true, because of the combinatorial factors to be introduced in order to restore matching between powers of η , so that, for instance, from the equation for \mathcal{J}' we find

$$\begin{aligned} \eta \sum_{k=1}^n \frac{1}{(2k-3)!!} \eta^{k-1} \varphi^{[k]} &= -\eta \frac{\rho_1}{m^2} \mathcal{J}', \\ \Rightarrow \\ \sum_{k=1}^n \frac{1}{(2k-3)!!} \eta^k \varphi^{[k]} &= -\eta \frac{\rho_1}{m^2} \mathcal{J}' + \sum_{k=2}^n \frac{1-k}{(2k-3)!!} \eta^k \varphi^{[k]}, \end{aligned} \quad (40)$$

which, in its turn, upon substitution in (38) gives

$$\mathcal{A} - m^2 \varphi = \mathcal{J} + \rho_1 \eta \mathcal{J}' + m^2 \sum_{k=2}^n \frac{k-1}{(2k-3)!!} \eta^k \varphi^{[k]}. \quad (41)$$

This observation suggests a quicker way to look for the solution of (38): rather than solving directly for the $\varphi^{[k]}$ in terms of $\mathcal{J}^{[k+1]}$, $\mathcal{J}^{[k+2]}$, \dots we shall substitute *the lines* of (39) in (38), taking care at each step of the corresponding remainder. Iterating this procedure one can prove by induction the following relation

$$\mathcal{A} - m^2 \varphi = \mathcal{J} + \sum_{k=1}^l \rho_k \eta^k \mathcal{J}^{[k]} + (-1)^{l+1} m^2 \sum_{k=l+1}^n \frac{\binom{k-1}{l}}{(2k-3)!!} \eta^k \varphi^{[k]}, \quad (42)$$

where for $l = n$ the remainder is not present, thus making it possible to identify the projection of the current giving rise to the massive propagator

$$\mathcal{A} - m^2 \varphi = \mathcal{J} + \eta \rho_1 (d-1, s) \mathcal{J}' + \eta^2 \rho_2 (d-1, s) \mathcal{J}'' + \dots + \eta^n \rho_n (d-1, s) \mathcal{J}^{[n]}, \quad (43)$$

¹³ For a discussion see [3].

consistently with (35).

It might be worth stressing that in the computation of the current interaction, under the assumption that the source be conserved, the structure of the Einstein tensor (7) plays no role, but for the main feature that it be divergenceless. This means that the detailed information about the coefficients of the projector obtained in (43) is entirely encoded in the form of the mass term (34), that receives in this way a non-trivial check on its correctness and uniqueness¹⁴. We expect the same mass term (34) to generate also consistent quadratic deformations of the massless geometric theory generalised to (A)dS backgrounds. The construction of this theory and the analysis of the corresponding current exchanges, to be compared with the results recently found in [3], are left for future work.

Acknowledgements I would like to thank J. Mourad and A. Sagnotti for discussions and collaboration. The present research was supported by the EU contract MRTN-CT-2004-512194.

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¹⁴ See [1], Sec. 4, for further considerations on the issue of uniqueness.